term for a unit which departs so much from the C. G. S. system. We agree entirely with the spirit of the recommendation adopted by the British Association for the Advancement of Science, 1898; Ostwald, 1899; and the International Congress of Physicists, Paris, 1900; all of whom appear to agree that the so-called "standard atmospheric pressure" (760 mm. of pure mercury under standard gravity at sea-level and latitude 45°) is not always the most appropriate datum for use.

In this connection we note that P. W. Bridgman (Phys. rev, Lancaster, Pa., (2), Feb., 1914, v. 3, p. 126, ffg) finds it convenient to use as his C. G. S. unit of pressure not dynes per square centimeter, but kilograms per square centimeter and the corresponding kilogram-meter per gram instead of gram-calory per gram.—[C. A.]



## THE C. G. S. SYSTEM AND METEOROLOGY.

By Prof. VILHELM BJERKNES, Leipzig.

[Translated from Meterologische Zeitschrift, Februar, 1913, p. 67–71.]

The International Commission for Scientific Aeronautics at its meeting in Vienna (1912) adopted the following resolution:

In the publications of the International Commission the pressure will be expressed in *bars* or in decimals thereof, such as *decibar*, *centibar*, *millibar*, instead of in millimeters of mercury; this decision will however first become effective when the International Meteorological Committee shall have communicated its agreement therewith.

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The principal advantages of the C. G. S. system were not considered during the discussion in Vienna, but were considered by all present as well known and recognized. But the subsequent discussion has shown that even on this point there prevails a surprising confusion. It will therefore not be improper to consider the question when we can apply entirely arbitrary units without injury, and when we can not relinquish the advantages of the C. G. S. system.

So long as scientific work consists only in the registration of individual elements and the statistical discussion of the resulting numerical series, we can without harm choose the units for the individual quantities quite arbitrarily—we merely need to apply the same units at various times and places; it is in this case quite unimportant whether the units thus applied to different quantities belong to a systematic system of units.

But so soon as we pass from climatological to dynamic researches we have to meet very different demands in order to understand the quantitative relations between the different quantities. For instance, we then no longer observe the pressure in order to consider the pressure itself, but in order to compute from it accelerations and velocities; we determine forces and motions not because of interest in these quantities themselves, but in order to compute from their combinations the work that is done and the heat that is evolved.

The conditions hitherto prevailing in meteorology have been very unfavorable for the development of this dynamic side of atmospherics. The equations of dynamics and of thermodynamics relate to the three dimensions of space and remain indefinite so long as we introduce into these equations only the results of observations obtained in two dimensions. With the establishment of aerology, these conditions have entirely changed. Simultaneous aerological observations give all the data needed for the direct application of the equations of dynamics and thermodynamics to meteorological problems, and thus open a prospect for an unsuspected development of meteorological science. But this development is restricted in its most sensitive portion as long as we retain an irrational unit of pressure for our simultaneous aerological observations. A single example will suffice to show the confusion that enters into dynamic equations as soon as we fail to apply a coherent system of units.

The condition of equilibrium in the atmosphere is as follows: The pressure against the boundary surfaces of any arbitrary volume of air must have a resultant that is directed vertically upward and is equal to the weight of the volume of air. If we consider a unit volume of this air, then its weight is equal to the product of its density $\rho$ into the acceleration of gravity $g$. The resultant of the pressures against the boundary surfaces of a unit of volume we call the pressure gradient $G$ (the dynamic definition of the gradient) and the equation of equilibrium takes the form

$$G = -\rho g \qquad \text{(a).}$$

The pressure gradient $G$ may also be defined simply as the change of pressure per unit of length which is the geometrical definition of the gradient. Therefore, if $z$ is a distance or length measured along the vertical, we have

$$G = -\frac{\partial p}{\partial z} \qquad \text{(b).}$$

Now (a) and (b) are the classic equations to which we are led under the condition that we are using a coherent system of units like the C. G. S. system.

But if we express the pressure in millimeters of mercury and retain the C. G. S. units for the density and the acceleration of gravity, then the equations (a) and (b) no longer harmonize but are incompatible with each other and clash together, and one or the other must be modified. If we decide to retain the geometric definition (b) for the gradient, then the equation of equilibrium (a) must be written in the form

$$1.333193\, G = -\rho g \qquad \text{(a}'\text{).}$$

The property of the gradients as simply equal numerically to the product of density and acceleration of gravity, is thus ignored.

If, on the other hand, we decide to hold fast to the dynamic definition of the gradient as in equation (a), then the geometric definition of this quantity must be expressed under the form

$$G = -1.333193\frac{\partial p}{\partial z} \qquad \text{(b}'\text{).}$$

In this equation the gradient loses its property of being equal to the negative of the change of pressure. Whichever way we may decide it is evident that we lose the simplicity and harmony of the systems of equations (a) and (b).

Confusions of this or a similar kind will be introduced into every dynamic or thermodynamic equation that contains the pressure, and uses millimeters of mercury as the unit of pressure while at the same time retaining the C. G. S. units for all other quantities. In order to realize the extent of this class of difficulties with which dynamic meteorology will be burdened so long as we continue to use the millimeter of mercury as the unit, it suffices to write out in full the equations that come into use in dynamic meteorology.

In these equations the independent variables are the time, $t$, and the coördinates $x$, $y$, $z$.

The components of the active forces are represented by $X$, $Y$, $Z$, which in the most general cases are compounded of the force of gravity, the deflective force of terrestrial rotation and the force of friction; their potential is $\Phi$.

The dependent variables are the following seven quantities:

$u$, $v$, $w$, the three components of the velocity of the air.

$p$, the pressure of the air.

$\rho$, the density of the air.

$\theta$, the absolute temperature of the air.

$r$, the moisture (i. e., vapor pressure per sq. cm.) of the air.

These seven dependent variables satisfy the seven equations that are written below. In Scheme I they are given in the classical form that they take when using the C. G. S. system. In Scheme II they are given in the form that they assume when we express the pressure in millimeters mercury, but retain the C. G. S. units for all other quantities.

The equations are as follows: The three hydrodynamic equations (1), (2), (3); the equation (4) of hydrodynamic continuity; the equation (5) of the gaseous condition; the equation (6) for the conservation of energy; the equation (7) that follows from the second law of thermodynamics. Besides the quantity of heat $dQ$, added to the mass of air under consideration, these two last equations contain two other new quantities, i. e., $E$, the energy, and $S$, the entropy, of the mass of air. The fact that these are known functions of the variables $p$, $\theta$, $r$, is expressed by the equations (6′) and (7′).

| SCHEME I. | SCHEME II. |
|---|---|
| $\rho\frac{du}{dt}=-\rho\frac{\partial\Phi}{\partial x}-\frac{\partial p}{\partial x}$ ..............(1) | $\rho\frac{du}{dt}=-\rho\frac{\partial\Phi}{\partial x}-1.333193\frac{\partial p}{\partial x}$ |
| $\rho\frac{dv}{dt}=-\rho\frac{\partial\Phi}{\partial y}-\frac{\partial p}{\partial y}$ ..............(2) | $\rho\frac{dv}{dt}=-\rho\frac{\partial\Phi}{\partial y}-1.333193\frac{\partial p}{\partial y}$ |
| $\rho\frac{dw}{dt}=-\rho\frac{\partial\Phi}{\partial z}-\frac{\partial p}{\partial z}$ ..............(3) | $\rho\frac{dw}{dt}=-\rho\frac{\partial\Phi}{\partial z}-1.333193\frac{\partial p}{\partial y}$ |
| $\frac{1}{\rho}\cdot\frac{d\rho}{dt}=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}$ ......(4) | $\frac{1}{\rho}\cdot\frac{d\rho}{dt}=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}$ |
| $\frac{p}{\rho}=R\theta$ ..........................(5) | $1.333193\frac{p}{\rho}=R\theta$ |
| $dQ=dE+pdv$ ..................(6) | $dQ=dE+1.333193pdv$ |
| $dS\gtrless 0$..........................(7) | $dS\gtrless 0$ |
| $E=f(p,\theta,r)$....................(6′) | $E=f(1.333193p,\theta,r)$ |
| $S=F\,(p,\theta,r)$ ..................(7′) | $S=F(1.333193p,\theta,r)$ |

These two systems of equations I and II differ from each other only in that no numerical factor enters the first set, whereas in the second set the numerical factor 1.333193 occurs everywhere in connection with the pressure. It is very evident that this factor causes an increase in the labor of computation. However, this inconvenience is only a small matter. The important objection to this second system of equations consists in the confusion of ideas introduced by this numerical factor. The nature of the confusion is illustrated by the above given example where we have considered the definition of the gradients. But the subject of this confusion is by far not exhausted by this one example; it recurs in innumerable varying forms, with every form of the equation.

It will not be possible to form clear plans for a fruitful coherent systematic development of observational and theoretical meteorology, unless we first consider carefully at every step how the above mentioned confusions are to be put aside or circumvented in the best way possible.

It must not be forgotten that the question here presented has an importance far beyond the limits of meteorology. The C. G. S. system has been planned as a universal system of units, and its universal application can not be prevented in the long run. At the present time we all regret that synoptic meteorology did not, at its very foundation, adopt the unit of pressure of this system. It is easily understood why at the present time and as conditions now exist, the general transfer of all meteorology to the C. G. S. system is delayed. The expenses and inconveniences that accompany the general transfer are very considerable, and the advantages will only be appreciated and become of great importance when the transfer has become really universal. Therefore it may still be proper to await the time when the British Empire and the United States shall have adopted the metric system.

But the conditions in regard to aerology are entirely different. This is a new branch of meteorology that is now in a stage of most rapid development and in which we can not afford to lose the benefit of the C. G. S. system, at least in the theoretical discussion of simultaneous ascensions of kites and balloons. It is of the greatest importance for the rational development of this branch that we allow ourselves the freedom of utilizing the advantages of the universal C. G. S. system.

## PROGRESS IN METEOROLOGICAL OPTICS DURING 1912.

By CHRISTIAN JENSEN, Hamburg.

[Translated by C. Abbe, jr., for the MONTHLY WEATHER REVIEW.]

During the year 1912 the occurrence that attracted the greatest attention was the general turbidity of the atmosphere. This unusual condition has shown itself in the low degree of saturation of blue skylight (1), the intense red coloring of the sun when near the horizon, the weakening of s arlight (2), the phenomenon of Bishop's Ring (3), and by other phenomena apparent to the naked eye. The most striking evidence, and of a quantitative character, was afforded by the instrumental measurements of the intensity of insolation and of sky polarization. We shall first review those publications of the year 1912 which deal with this atmospheric turbidity, but it will sometimes be necessary to touch upon studies made during the year 1913 also.

It is generally agreed that the tremendous explosions of the volcano Katmai in Alaska from June 6 to June 9, 1912, produced the exciting cause of that optical disturbance whose principal effects over Europe began on June 20 of that year. But as we have already pointed out (4) the fact must not be overlooked that reports of antecedent optical disturbances at various points, indicate that there was a preexistant condition of general turbidity. Thus the well-known investigator of the zodiacal light, Schmid (5) of Oberhelfenswil, reported that in the second half of May the sun was the center of a peculiar silver-white disk 8° to 10° in diameter; F. Hahn (6) reports that as early as June 8 he noticed the peculiar

[1] From Mitteilungen der Vereinigung von Freunden der Astronomie und kosmischen Physik, Berlin, 1913, pp. 166-183.